UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

MATHEMATICS (SYLLABUS D) **4024/02**

Paper 2

May/June 2004

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)

READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

This document consists of **11** printed pages and **1** blank page.

SP (NF/GR) S64038/4

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

Section A [52 marks]

Answer **all** the questions in this section.

1 Two villages, P and Q, are joined by a straight road 6000 m long.

(a) Ann left P and ran to Q at a steady speed of 3 m/s.
At the same instant that Ann left P, Ben left Q and cycled to P at a steady speed of 7 m/s.

(i) **(a)** How far, in metres, did Ann travel in the first 2 minutes? [1]

(b) Calculate the distance between Ann and Ben at the end of the first 2 minutes. [1]

(ii) Ann and Ben passed each other at M.

Calculate the distance PM. [2]

(iii) Calculate the time that Ben took to cycle from Q to P.
Give your answer in minutes and seconds, correct to the nearest second. [2]

(b) The villages appear on a map which has a scale of 2 cm to 5 km.

(i) Express this scale in the form 1 : n. [1]

(ii) Calculate the length of the road joining P and Q on the map. [2]

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2 **(a)** Factorise completely $2tv + t - 10v - 5$. [2]

(b) Make k the subject of the formula

$$\sqrt{\frac{h}{k}} = 3.$$

[2]

(c) Solve the equation $x^2 - 23x + 81 = 0$, giving both answers correct to two decimal places. [4]

(d) The matrix **Y** satisfies the equation

$$4\mathbf{Y} - 2\begin{pmatrix} 12 & 6 \\ -9 & 0 \end{pmatrix} = \mathbf{Y}.$$

Find **Y**, expressing it in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. [2]

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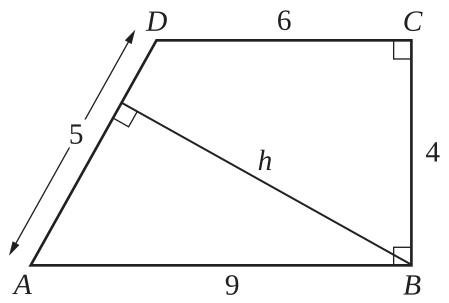
3 **(a)** The diagram shows a trapezium *ABCD*.

Angle *ABC* and angle *BCD* are right angles.

AB = 9 cm, *BC* = 4 cm, *CD* = 6 cm and *DA* = 5 cm.

The perpendicular distance from *B* to *AD* is *h* centimetres.



Calculate

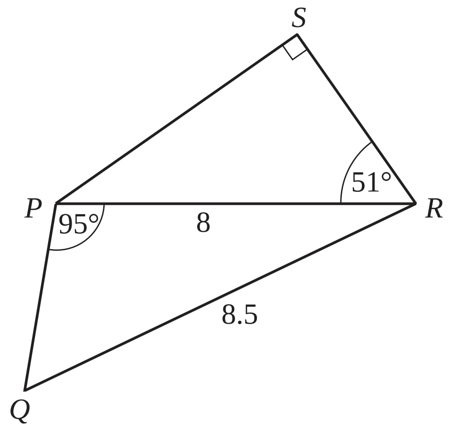
(i) the area of the trapezium, [1]

(ii) the value of *h*, [2]

(iii) angle *DAB*. [2]

(b) The diagram shows two triangles, *PRS* and *PRQ*.

$PR = 8$ cm, $QR = 8.5$ cm, $P\hat{S}R = 90°$, $P\hat{R}S = 51°$ and $R\hat{P}Q = 95°$.



(i) Calculate *RS*. [2]

(ii) Calculate $P\hat{Q}R$. [3]

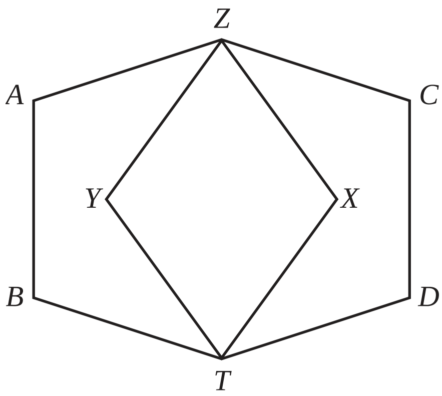
(iii) A circle is drawn through *P*, *R* and *S*.

(a) Does this circle pass through *Q*?
Give a reason for your answer. [1]

(b) Where is the centre of this circle? [1]

4 **(a)** Show that the interior angle of a regular pentagon is 108°. [2]

(b)



The diagram shows two congruent, regular pentagons, *ZABTX* and *ZCDTY*.

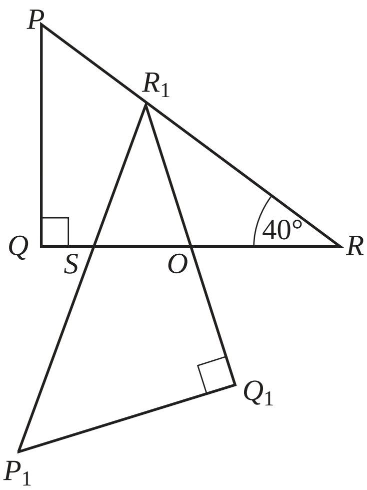
(i) Describe fully all the symmetries of this diagram. [2]

(ii) What is the special name given to the quadrilateral *ZXTY*? [1]

(iii) Calculate **reflex** angle *ZYT*. [1]

(iv) Calculate angle *AZY*. [1]

(c)



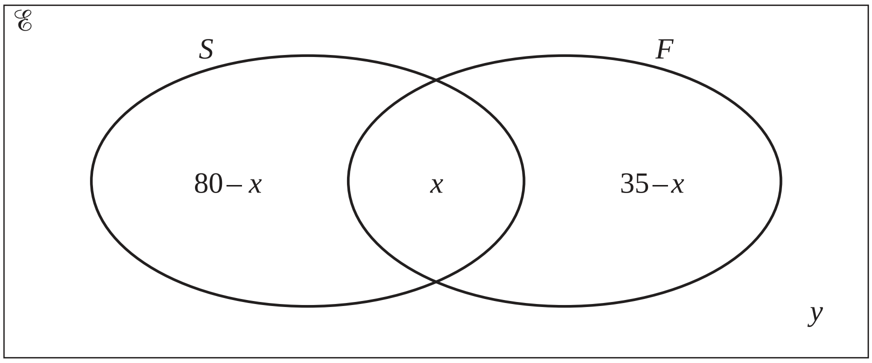
In the diagram, *PQR* is a triangle with $P\hat{Q}R$ = 90° and $Q\hat{R}P$ = 40°.
The point *O* is the midpoint of *QR*.
Triangle $P_1Q_1R_1$ is the image of triangle *PQR* under an anticlockwise rotation about the point *O*.
The point R_1 lies on *PR*.
The line *QR* intersects the line P_1R_1 at the point *S*.
Find

(i) $R\hat{R}_1Q_1$, [1]

(ii) the angle of rotation , [1]

(iii) $O\hat{S}P_1$. [1]

5 In a group of 100 students, 80 study Spanish and 35 study French.
x students study Spanish and French.
y students study neither Spanish nor French.
The Venn diagram illustrates this information.



(a) Expressed in set notation, the value of x is $n(S \cap F)$.
Express the value of y in set notation. [1]

(b) Find, in its simplest form, an expression for y in terms of x. [2]

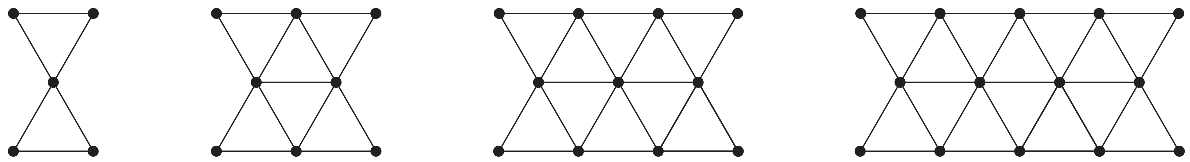
(c) Find

(i) the least possible value of x, [1]

(ii) the greatest possible value of y. [1]

6 Bob makes fences using identical metal rods one metre long.
The rods are bolted together at their ends.

Some fences, with different lengths, are shown below.



Length = 1 m   Length = 2 m   Length = 3 m   Length = 4 m

• shows the position of a bolt.

The table shows the numbers of bolts and rods used for various lengths of fence.

Length (metres)	1	2	3	4	.....	n
Number of bolts	5	8	11	p	.....	B
Number of rods	6	13	20	q	.....	R

(a) Write down the values of p and q. [1]

(b) Given that $B = 3n + k$, where k is a constant, find the value of k. [1]

(c) Find an expression for R in terms of n. [2]

(d) Bob has 200 bolts and 400 rods.

How many complete fences can he make which have a length of 6 m? [2]

Section B [48 marks]

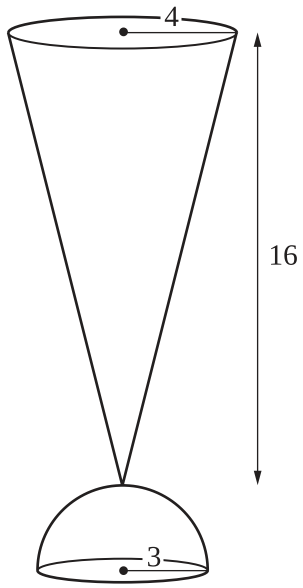
Answer **four** questions in this section.

Each question in this section carries 12 marks.

7 [The surface area of a sphere is $4\pi r^2$.]

[The volume of a cone is $\frac{1}{3} \times$ base area $\times$ height.]

[The area of the curved surface of a cone of radius r and slant height l is πrl.]



A drinking glass consists of a hollow cone attached to a solid hemispherical base as shown in the diagram.
The hemisphere has a radius of 3 cm.
The radius of the top of the cone is 4 cm and the height of the cone is 16 cm.

(a) Calculate the **total** surface area of the solid hemispherical base. [3]

(b) Calculate the curved surface area of the outside of the cone. [3]

(c) **(i)** The cone contains liquid to a depth of d centimetres.

Giving your reasons, show that the radius of the surface of the liquid is $\frac{1}{4}d$ centimetres. [1]

(ii) The cone is completely filled with liquid.

Calculate the volume of the liquid. [2]

(iii) Half of the volume of the liquid from the full cone is now poured out.

Using the answers to parts **(i)** and **(ii)**, find the depth of the liquid that remains in the cone. [3]

8 **Answer the whole of this question on a sheet of graph paper.**

The table gives the x and y coordinates of some points which lie on a curve.

x	1	1.5	2	2.5	3	4	5	6
y	140	110	100	98	100	110	124	140

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 6$.

Using a scale of 2 cm to represent 10 units, draw a vertical y-axis for $90 \leqslant y \leqslant 150$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find

(i) the value of y when $x = 4.5$, [1]

(ii) the values of x for which $y = 128$. [1]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

(d) The line $y = k$ is a tangent to the curve.

Find the value of k. [1]

(e) The values of x and y are related by the equation

$$y = \frac{A}{x} + Bx.$$

(i) Use the fact that the point (2, 100) lies on the curve to show that

$$200 = A + 4B.$$ [1]

(ii) Obtain a second equation connecting A and B.

Hence calculate the value of A and the value of B. [3]

9

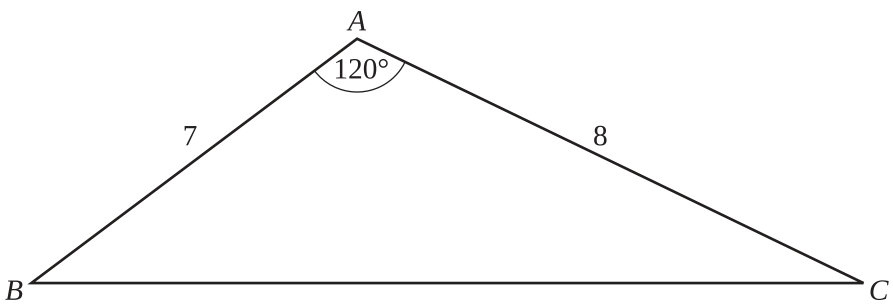


Diagram I

Diagram I shows a triangle ABC in which $AB = 7$ cm, $AC = 8$ cm and $B\hat{A}C = 120°$.

(a) Show that $BC = 13$ cm. [2]

(b) Calculate the area of triangle ABC. [2]

(c)

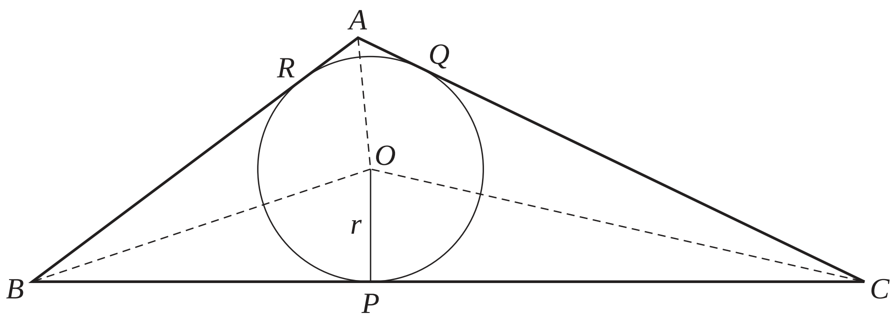


Diagram II

The sides of the triangle ABC, shown in Diagram I, are tangents to a circle with centre O and radius r centimetres.
The circle touches the sides BC, CA and AB at P, Q and R respectively, as shown in Diagram II.

(i) Find an expression, in terms of r, for the area of triangle OBC. [1]

(ii) By similarly considering the areas of triangles OAB and OAC, find an expression, in terms of r, for the area of triangle ABC. [2]

(iii) Hence find the value of r. [2]

(d) Calculate the percentage of the area of triangle ABC that is **not** occupied by the circle. [3]

10 Answer the whole of this question on a sheet of graph paper.

The ages of a sample of 40 students were recorded.
The results are given in the table below.

Age (x years)	$8 < x \leqslant 10$	$10 < x \leqslant 11$	$11 < x \leqslant 12$	$12 < x \leqslant 14$	$14 < x \leqslant 16$	$16 < x \leqslant 19$
Frequency	7	8	6	10	3	6

(a) Using a scale of 1 cm to represent 1 year, draw a horizontal axis for ages from 8 to 19 years.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 8 units.
On your axes, draw a histogram to illustrate the distribution of ages. [3]

(b) In which interval does the median lie? [1]

(c) Calculate an estimate of the mean age of the students. [3]

(d) Calculate an estimate of the number of students who were under 13 years old. [1]

(e) One student is chosen at random from this sample of 40 students.

Write down the probability that this student is

(i) under 8, [1]

(ii) over 16. [1]

(f) A second student is now chosen at random from the remaining 39 students.

Calculate the probability that one student is over 16 and the other is not over 16.
Give your answer as a fraction in its lowest terms. [2]

11 **(a)** In a swimming match between two schools, *C* and *D*, two students from each school took part in each event.
The number of places each school gained in each position is shown in the table.

	First	Second	Third	Fourth
School *C*	6	3	5	6
School *D*	4	7	5	4

The points awarded for First, Second, Third and Fourth places were 5, 3, 1 and 0 respectively.

Matrices related to this information are defined below.

$$\mathbf{A} = \begin{pmatrix} 6 & 3 & 5 & 6 \\ 4 & 7 & 5 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 5 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

(i) What does the sum of the elements in each column of **A** represent? [1]

(ii) **(a)** Find **AB**. [2]

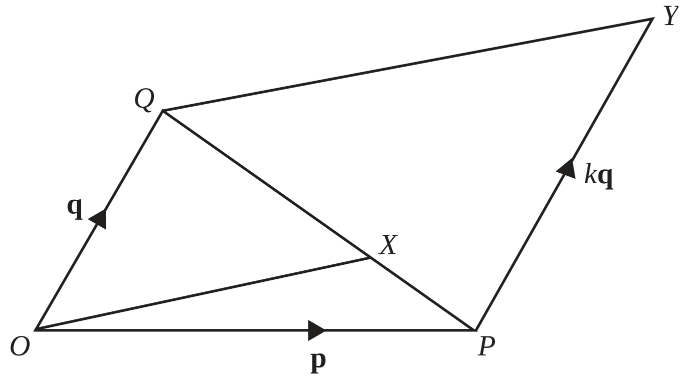
(b) What information is shown by **AB**? [1]

(iii) It was suggested that the points awarded for First, Second, Third and Fourth places should have been 5, 3, 2 and 1 respectively.

Would this suggestion have made any difference to which school won this match?
Show clear working to justify your answer. [1]

(b) In the diagram,

$\overrightarrow{OP} = \mathbf{p}$,
$\overrightarrow{OQ} = \mathbf{q}$,
$\overrightarrow{PY} = k\mathbf{q}$,
$\overrightarrow{PX} = \frac{1}{3}\overrightarrow{PQ}$.



(i) Express $\overrightarrow{PX}$ in terms of **p** and **q**. [1]

(ii) Express $\overrightarrow{OX}$ in terms of **p** and **q**. [1]

(iii) Express $\overrightarrow{QY}$ in terms of k, **p** and **q**. [1]

(iv) Given that OX is parallel to QY, find the value of k. [2]

(v) The line OX, when produced, meets PY at Z.

Express $\overrightarrow{PZ}$ in terms of **q**. [2]

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